NAG Toolbox for MATLAB

g05hd

1 Purpose

g05hd generates a realization of a multivariate time series from a vector autoregressive moving average (VARMA) model. The realization may be continued or a new realization generated at subsequent calls to g05hd.

2 Syntax

```
[w, ref, ifail] = g05hd(mode, ip, iq, mean, par, qq, n, ref, 'k', k,
'lpar', lpar, 'lref', lref)
```

3 Description

Let the vector $W_t = (w_{1t}, w_{2t}, \ldots, w_{kt})^{\mathrm{T}}$ denote a k-dimensional time series which is assumed to follow a vector autoregressive moving average (VARMA) model of the form:

$$\begin{aligned} W_t - \mu = \; & \phi_1(W_{t-1} - \mu) + \phi_2(W_{t-2} - \mu) + \cdots + \phi_p(W_{t-p} - \mu) \\ & + \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2} - \cdots - \theta_q\epsilon_{t-q} \end{aligned} \tag{1}$$

where $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t}, \ldots, \epsilon_{kt})^{\mathrm{T}}$ is a vector of k residual series assumed to be Normally distributed with zero mean and positive-definite covariance matrix Σ. The components of ϵ_t are assumed to be uncorrelated at non-simultaneous lags. The ϕ_is and θ_js are k by k matrices of parameters. $\{\phi_i\}$, for $i = 1, 2, \ldots, p$, are called the autoregressive (AR) parameter matrices, and $\{\theta_j\}$, for $j = 1, 2, \ldots, q$, the moving average (MA) parameter matrices. The parameters in the model are thus the pk by $k\phi$-matrices, the qk by $k\theta$-matrices, the mean vector μ and the residual error covariance matrix Σ. Let

$$A(\phi) = \begin{bmatrix} \phi_1 & I & 0 & . & . & . & 0 \\ \phi_2 & 0 & I & 0 & . & . & 0 \\ . & & & . & & & \\ . & & & & . & & \\ . & & & & & . & \\ \phi_{p-1} & 0 & . & . & . & 0 & I \\ \phi_p & 0 & . & . & . & 0 & 0 \end{bmatrix}_{pk \times pk} \quad \text{and} \quad B(\theta) = \begin{bmatrix} \theta_1 & I & 0 & . & . & . & 0 \\ \theta_2 & 0 & I & 0 & . & . & 0 \\ . & & & . & & & \\ . & & & & . & & \\ . & & & & & . & \\ \theta_{q-1} & 0 & . & . & . & 0 & I \\ \theta_q & 0 & . & . & . & 0 & 0 \end{bmatrix}_{qk \times qk}$$

where I denotes the k by k identity matrix.

The model (1) must be both stationary and invertible. The model is said to be stationary if the eigenvalues of $A(\phi)$ lie inside the unit circle and invertible if the eigenvalues of $B(\theta)$ lie inside the unit circle.

For $k \geq 6$ the VARMA model (1) is recast into state space form and a realization of the state vector at time zero computed. For all other cases the function computes a realization of the pre-observed vectors $W_0, W_{-1}, \ldots, W_{1-p}$, $\epsilon_0, \epsilon_{-1}, \ldots, \epsilon_{1-q}$, from equation (1), see Shea 1988. This realization is then used to generate a sequence of successive time series observations. Note that special action is taken for pure MA models, that is for $p = 0$.

At your request a new realization of the time series may be generated with less computation using only the information saved in a reference vector from a previous call to g05hd. See the description of the parameter **mode** in Section 5 for details.

The function returns a realization of $W_1, W_2, \ldots, W_n$. On a successful exit, the recent history is updated and saved in the array **ref** so that g05hd may be called again to generate a realization of $W_{n+1}, W_{n+2}, \ldots$, etc. See the description of the parameter **mode** in Section 5 for details.

Further computational details are given in Shea 1988. Note however that this function uses a spectral decomposition rather than a Cholesky factorization to generate the multivariate Normals. Although this method involves more multiplications than the Cholesky factorization method and is thus slightly slower it is more stable when faced with ill-conditioned covariance matrices. A method of assigning the AR and MA coefficient matrices so that the stationarity and invertibility conditions are satisfied is described in Barone 1987.

4 References

Barone P 1987 A method for generating independent realisations of a multivariate normal stationary and invertible ARMA(p, q) process *J. Time Ser. Anal.* **8** 125–130

Shea B L 1988 A note on the generation of independent realisations of a vector autoregressive moving average process *J. Time Ser. Anal.* **9** 403–410

5 Parameters

5.1 Compulsory Input Parameters

1: **mode – string**

Must be set as follows:

mode = 'S' (**S**tart)

The function is being called for the first time; a realization of the recent history is computed, and the sequence of time series values from the VARMA model is then generated.

mode = 'R' (**R**estart)

The function must have been called before with the same VARMA model; a new realization of the recent history is computed using information stored in the reference vector, followed by the sequence of time series values.

mode = 'C' (**C**ontinue)

The function must have been called before with the same VARMA model; a new sequence is generated, from the point at which the last sequence ended, using a realization of the recent history which was updated and stored by the previous call to the function.

mode = 'R' or 'C'

You must ensure that the reference vector **ref** and the values of **k**, **ip**, **iq**, **mean**, **par**, **qq** and **ldqq** have not been changed between calls to g05hd.

Constraint: **mode** = 'S', 'R' or 'C'.

2: **ip – int32 scalar**

p, the number of AR parameter matrices.

Constraint: **ip** ≥ 0.

3: **iq – int32 scalar**

q, the number of MA parameter matrices.

Constraint: **iq** ≥ 0.

4: **mean – string**

Indicates whether or not all elements of μ are to be supplied by you or to be taken as zero.

mean = 'M'

The values of μ, are supplied in the array **par**.

mean = 'Z'

All elements of μ are to be taken as zero.

Constraint: **mean** = 'M' or 'Z'.

5: **par(lpar) – double array**

μ, the parameter values read in row by row in the order $\phi_1, \phi_2, \ldots, \phi_p$, $\theta_1, \theta_2, \ldots, \theta_q$.

If **ip** > 0, then **par**$((l-1) \times k \times k + (i-1) \times k + j)$ must be set equal to the (i,j)th element of ϕ_l, for $l = 1, 2, \ldots, p$ and $i,j = 1, 2, \ldots, k$.

If **iq** > 0, then **par**$(p \times k \times k + (l-1) \times k \times k + (i-1) \times k + j)$ must be set equal to the (i,j)th element of θ_l, for $l = 1, 2, \ldots, q$ and $i,j = 1, 2, \ldots, k$.

If **mean** = 'M', then **par**$((p+q) \times k \times k + i)$ must be set equal to the ith component of the mean vector μ, for $i = 1, 2, \ldots, k$.

Constraint: the first **ip** × **k** × **k** elements of **par** must satisfy the stationarity condition and the next **iq** × **k** × **k** elements of **par** must satisfy the invertibility condition.

6: **qq(ldqq,k) – double array**

ldqq, the first dimension of the array, must be at least **k**.

qq(i,j) must contain the (i,j)th element of Σ. Only the lower triangle is required.

Constraint: the elements of **qq** must be such that Σ is positive-definite.

7: **n – int32 scalar**

n, the number of observations to be generated.

Constraint: **n** ≥ 1.

8: **ref(lref) – double array**

If **mode** = 'R' or 'C', then the array **ref** as output from the previous call to g05hd must be input without any change to the first $m + (k+1)(k+2) + (m+1)(m+2)$ elements where $m = k \times \max(p,q)$ if $k \geq 6$ and $k(p+q)$ if $k < 6$.

If **mode** = 'S', then the contents of **ref** need not be set.

5.2 Optional Input Parameters

1: **k – int32 scalar**

Default: The dimension of the array **qq**.

Constraint: **k** ≥ 1.

2: **lpar – int32 scalar**

Default: The dimension of the array **par**.

Constraints: **lpar** $\geq \max(1, npar)$, where

$npar = (\mathbf{ip} + \mathbf{iq}) \times \mathbf{k} \times \mathbf{k}$ if **mean** = 'Z' and

$npar = (\mathbf{ip} + \mathbf{iq}) \times \mathbf{k} \times \mathbf{k} + \mathbf{k}$ if **mean** = 'M'.

3: **lref – int32 scalar**

Default: The dimension of the array **ref**.

Constraints: $r = \max(\mathbf{ip}, \mathbf{iq})$ and

$l = \mathbf{k}(\mathbf{k}+1)/2$ if $\mathbf{ip} = 0$,
$l = \mathbf{k}(\mathbf{k}+1)/2 + (\mathbf{ip}-1)\mathbf{k}^2$ if $\mathbf{ip} \geq 1$.

If $\mathbf{k} \geq 6$, then $\mathbf{lref} \geq \left(5r^2+1\right)\mathbf{k}^2 + (4r+3)\mathbf{k} + 4$.

If $\mathbf{k} < 6$, then $\mathbf{lref} \geq \left((\mathbf{ip}+\mathbf{iq})^2+1\right)\mathbf{k}^2+(4(\mathbf{ip}+\mathbf{iq})+3)\mathbf{k}+\max\{\mathbf{k}r(\mathbf{k}r+2),$
$\mathbf{k}^2(\mathbf{ip}+\mathbf{iq})^2+l(l+3)+\mathbf{k}^2(\mathbf{iq}+1)\}+4.$

See Section 8 for some examples of the required size of the array **ref**.

5.3 Input Parameters Omitted from the MATLAB Interface

ldqq, iwork, liwork

5.4 Output Parameters

1: **w(ldqq,n) – double array**

$\mathbf{w}(i,t)$ will contain a realization of the ith component of W_t, for $i = 1,2,\ldots,k$ and $t = 1,2,\ldots,n$.

2: **ref(lref) – double array**

The first $m+(k+1)(k+2)+(m+1)(m+2)$ elements of the array **ref** contain information required for any subsequent calls to the function with **mode** = 'R' or 'C'; the rest of the array is used as workspace. See Section 8.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **mode** $\neq$ 'S', 'R' or 'C',
or $\mathbf{k} < 1$,
or $\mathbf{ip} < 0$,
or $\mathbf{iq} < 0$,
or **mean** $\neq$ 'M' or 'Z',
or $\mathbf{lpar} < \max((\mathbf{ip}+\mathbf{iq})\times\mathbf{k}\times\mathbf{k}+\mathbf{k},1)$ and **mean** = 'M',
or $\mathbf{lpar} < \max((\mathbf{ip}+\mathbf{iq})\times\mathbf{k}\times\mathbf{k},1)$ and **mean** = 'Z',
or $\mathbf{ldqq} < \mathbf{k}$,
or $\mathbf{n} < 1$,
or **lref** is too small,
or **liwork** is too small.

ifail = 2

On entry, either the value of Σ is not positive-definite, or the AR parameters are such that the model is non-stationary, or the MA parameters are such that the model is non-invertible. To proceed, you must try different parameter values.

ifail = 3

This is an unlikely exit brought about by an excessive number of iterations being needed by the function used to evaluate the eigenvalues of $A(\phi)$ or $B(\theta)$. If this error occurs please contact NAG.

ifail = 4

g05hd has not been able to calculate all the required elements of the array **ref**. This is an unlikely exit brought about by an excessive number of iterations being needed by the function to evaluate eigenvalues to be stored in the array **ref**. If this error occurs please contact NAG.

ifail = 5

g05hd has not been able to calculate all the required elements of the array **ref**. This is likely to be because the AR parameters are very close to the boundary of the stationarity region.

ifail = 6

The reference vector **ref** has been corrupted, when **mode** is set to 'R' or 'C'. To proceed, you should set **mode** to 'S'.

7 Accuracy

The accuracy is limited by the matrix computations performed, and this is dependent on the condition of the parameter and covariance matrices.

8 Further Comments

Note that, in reference to **ifail** = 2, g05hd will permit MA parameters on the boundary of the invertibility region.

The elements of **ref** contain amongst other information details of the spectral decompositions which are used to generate future multivariate Normals. Note that these eigenvectors may not be unique on different machines. For example the eigenvectors corresponding to multiple eigenvalues may be permuted. Although an effort is made to ensure that the eigenvectors have the same sign on all machines, differences in the signs may theoretically still occur.

The following table gives some examples of the required size of the array **ref**, specified by the parameter **lref**, for $k = 1, 2, 3$, and for various values of p and q.

		q			
		0	1	2	3
p	0	13 36 85	20 56 124	31 92 199	46 144 310
	1	19 52 115	30 88 190	45 140 301	64 208 448
	2	35 136 397	50 188 508	69 256 655	92 340 838
	3	57 268 877	76 336 1024	99 420 1207	126 520 1426

Note that g13dx may be used to check whether a VARMA model is stationary and invertible.

The time taken depends on the values of p, q and especially n and k.

9 Example

```
mode = 'Start';
ip = int32(1);
iq = int32(0);
mean = 'M';
par = [0.8;
```

```
     0.07;
     0;
     0.58;
     5;
     9];
qq = [2.97, 0;
     0.64, 5.38];
n = int32(48);
ref = zeros(553, 1);
g05za('O');
g05cb(int32(0));
[w, refOut, ifail] = g05hd(mode, ip, iq, mean, par, qq, n, ref)
```

```
w =
  Columns 1 through 7
    9.7225   11.1005    8.7070    7.5011    7.7570   12.1426    9.7519
   10.4340    8.2327    9.4026    5.8380   10.6743   10.2411    9.5962
  Columns 8 through 14
   10.6240   10.1970    8.5957    9.7515    9.4407    9.7329    9.9702
    6.8134    8.3194    5.9211    8.2144   13.6182   12.4765   11.4535
  Columns 15 through 21
   10.5845    9.8202    9.8471    6.4138    5.5480    6.2124    5.2033
   13.7748    9.4509   12.9020    8.9834    5.3801    7.5105    4.5219
  Columns 22 through 28
    3.9340    3.0080    3.2346    4.0515    4.6891   10.8085    7.6489
    3.3858    3.7349    9.2746    7.6747    3.8875    7.0680    7.0109
  Columns 29 through 35
   11.3451    8.5221    8.9819    7.3938    4.0279    3.1608    2.7069
    6.9254    5.2897    5.7948    3.7954    4.1429    8.2686   10.3505
  Columns 36 through 42
    3.6961    2.4295    4.5526    7.3014    3.0897    2.8447    4.6246
    6.2802    8.8904    8.8388   10.9436   11.2194    9.3870   11.2665
  Columns 43 through 48
    6.7365    8.1938    7.2362    8.5037    9.1628   10.5620
   11.0489    9.2139    9.6382    8.9742    8.1785   10.7348
refOut =
     array elided
ifail =
           0
```